Section 3-2, Mathematics 108

Polynomial Functions

Graphing a polynomial function of degree > 2 has some complications that are not seen with quadratics.

It is easy now to get a good picture quite easily using a graphing program.
Even so, there are some general features that are good to understand.

End Behavior

By end behavior we mean, what happens as *x* gets very large or very negatively large. In all cases the **leading** term of the polynomial will dominate because it raises *x* to the highest power.

In all cases the end behavior of the function is to either get very large or very negatively large. Which of these it does depends on the leading coefficient and whether the degree is even or not.

	Degree Even	Degree Odd
Leading Coefficient > 0	$x \to \infty \quad f(x) \to \infty$ $x \to -\infty \quad f(x) \to \infty$	$x \to \infty \quad f(x) \to \infty$ $x \to -\infty \quad f(x) \to -\infty$
Leading Coefficient < 0	$x \to \infty \quad f(x) \to -\infty$ $x \to -\infty \quad f(x) \to -\infty$	$x \to \infty \quad f(x) \to -\infty$ $x \to -\infty \quad f(x) \to \infty$

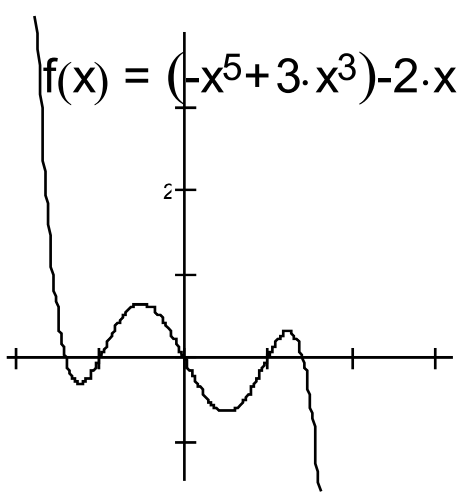
Example:

$f(x) = -x^5 + 3x^3 - 2x$

Since this is degree 5, and odd number and the leading coefficient is negative we would expect

$x \to \infty \quad f(x) \to -\infty$

$x \to -\infty \quad f(x) \to \infty$



Zeros of a Polynomial Function

A polynomial function if degree *n* can have at most real *n* zeros.

Here are four equivalent statements about a polynomial *P*(*x)* function's real zeros.

1. *c* is a zero of P

2. $x=c$ is a solution of the equation $P(x) = 0$

3. $(x\text{-}c)$ is a factor of $P(x)$.

4. *c* is an *x*-intercept of the graph of *P*.

Intermediate Value Theorem for Polynomials

The graphs of all polynomials are smooth and continuous without breaks.
So if you have two values *a* and *b* with $f(a) > 0$ and $f(b) < 0$
then there is some c between *a* and *b* such that $P(c) = 0$.

This is called the Intermediate Value Theorem.

Actually the intermediate value theorem is more general. It says that if

if $a \neq b$ and $f(a) < f(b)$ then for all $y \mid f(a) < y < f(b)$ there exists at least one value *c* such that $y = f(c)$.

Local Maximum and Minimum values for a Polynomial

For any two zeros $a < b$ of a polynomial, there exists either a local minimum or local maximum on the interval [*a,b*].

Since a polynomial of degree *n* has at most *n* roots, it will have at most *n*-1 local minimum or maximums between them.

All of this information can be useful when graphing.

Example:

$$f(x) = x^3 - 2x^2 - 3x$$

We know that for x large this will go to infinity and for x negative this will go to negative infinity.

Factoring we get

$$f(x) = x(x+1)(x-3)$$

So we know there are zeros at 0, -1 and 3.

Since x much less than -1 is very negative, between -1 and 0 there will be a local maximum.

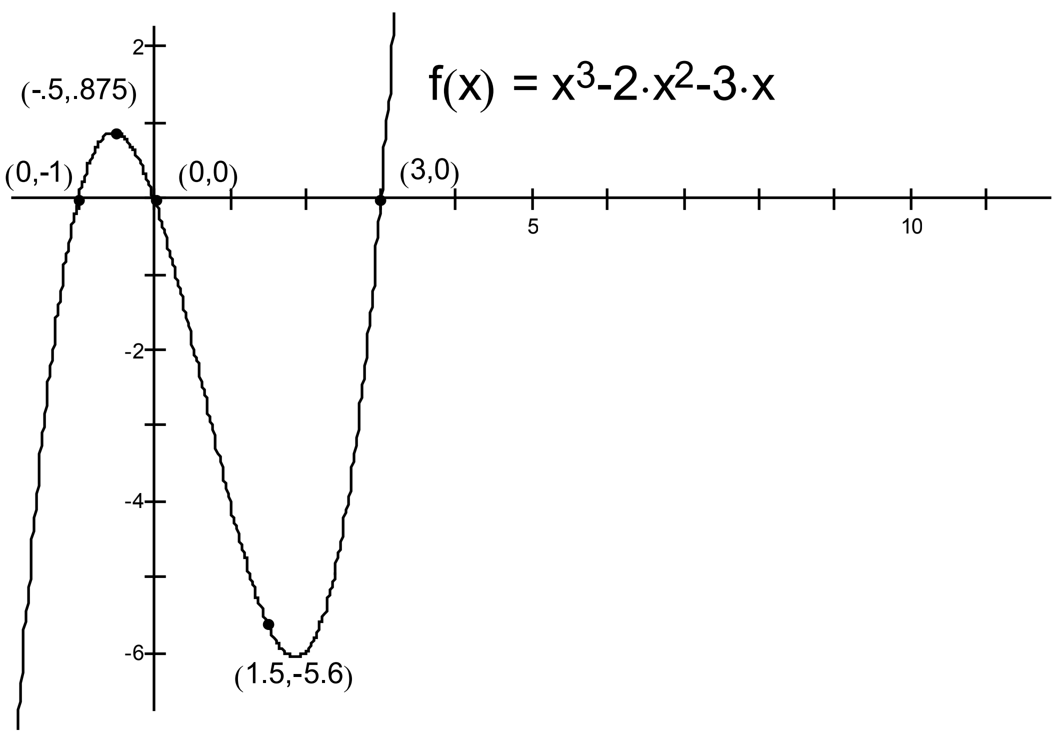
Also between 0 and 3 there will be a local minimum.

Plotting the points

$$f(-.5) = .875$$

$$f(1.5) = -5.6$$

We get a pretty good idea what the graph looks like



Duplicate Roots

Sometimes a polynomial will have the same root more than once.

Two examples are

$$f(x) = x^2$$

$$f(x) = x^3$$

The first has root x=0 twice and the second has root x=0 three times.

The important thing to note is that when a root has an even number of the same root, it will not pass through the X-axis, whereas if it has an odd number, it will.

Example:

$$f(x) = x^4 (x-2)^3 (x+1)^2$$

Looking at this function we can see that it has a root at zero that will not cross the axis, a root at 2 that will, and one at -1 that will not. Here is what the function looks like: